

JOINT APPRENTICESHIP & TRAINING
COMMITTEE OF PIPE FITTERS LOCAL UNION 211

STUDY GUIDE

PLEASE MAKE SURE YOU HAVE ALL REQUIRED
DOCUMENTS BEFORE CALLING TO SCHEDULE AN
APPOINTMENT FOR TESTING. (713) 649-0201

YOU HAVE ONE HOUR TO TEST. NO CALCULATORS OR CELL PHONES
ARE ALLOWED IN THE TEST ROOM.

THE TEST CAN BE TAKE UP TO 3 TIMES.

AN OSHA 10 IS REQUIRED- YOU WILL BE PROVIDED WITH THE
INFORMATION TO AQUIRE AN OSHA 10, AFTER YOU PASS THE TEST.

INTRODUCTION TO WHOLE NUMBER STUDY SHEET

ADDITION: The process of finding the total or sum of 2 or more numbers.

SIGN: + or plus or add.

EXAMPLE:

Tens ----->	85
Hundreds ----->	664
Thousands ----->	7,563
Sign ----->	<u>+ 4</u> – Units
	8,316 ----- Sum or Total

PROBLEM:

614	312
03	6,984
<u>+ 1,498</u>	Add: <u>21</u>

798	2
211	100
390	290
100	311
2	608
<u>+ 603</u>	Plus: <u>793</u>

INTRODUCTION TO WHOLE NUMBERS
STUDY SHEET

SUBTRACTION: The process of finding the difference between 2 numbers.

SIGN: - or minus or subtract

EXAMPLE: 529
 - 156
 373

TO CHECK ANSWER: 156
 + 373
 529

PROBLEM: 309 631
 - 154 - 28

 628 7,315
 - 31 - 6,429

INTRODUCTION TO WHOLE NUMBERS
STUDY SHEET

DIVISION: The process of finding how many times one number is contained within another.

SIGN: \div or $\overline{)}$

EXAMPLE: $\begin{array}{r} 2 \\ 3\overline{)6} \end{array}$ $\begin{array}{r} 4 \\ 4\overline{)16} \end{array}$ $\begin{array}{r} 20 \\ 9\overline{)180} \end{array}$

WRITTEN EXAMPLE: Six \div Three All of these
Six divided by three have the
Six \div Three same meaning

PROBLEM: 25 divided by five

Eighteen \div 6

$7\overline{)42}$

$9\overline{)81}$

INTRODUCTION TO FRACTIONS STUDY SHEET

FRACTION: This term is used mathematically to indicate that a number is less than a whole.

EXAMPLE: A half dollar is a part of one whole dollar. One half is written $\frac{1}{2}$, meaning the whole (1) has been divided into 2 equal parts. Thus $\frac{1}{2}$.

NUMERATORS AND DENOMINATORS: Two numbers must be used to state a fraction. The upper is called the numerator and the lower is called the denominator. The denominator indicates the number of times the whole number has been divided. The numerator indicates the number of parts under discussion.

EXAMPLE: $\frac{3}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{16}$

These are all Proper Fractions because they all are less than 1. (The numerator is smaller than the denominator.)

INTRODUCTION TO FRACTIONS STUDY SHEET

IMPROPER FRACTIONS: A fraction that is equal to one or more than one is called an improper fraction.

EXAMPLE: $6/6$ $4/3$ $3/2$ $16/7$ $5/8$

These are all equal to one or more than one.

MIXED NUMBER: A number made up of a whole number and a fraction is called a mixed number.

$1 \frac{3}{4}$ $2 \frac{7}{8}$ $9 \frac{1}{3}$ These all represent a whole number plus part of another number.

REDUCING TO THE LOWEST TERMS: A fraction is usually expressed in its lowest term. Whenever the numerator and denominator can be divided evenly by the same number the fraction can be reduced.

EXAMPLE: $5/10$ both of these numbers may be divided by 5

THUS:
$$\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

OR:
$$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

INTRODUCTION TO FRACTIONS STUDY SHEET

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS:

Multiplication and division are required to change a mixed number into an improper fraction.
3 steps are required.

EXAMPLE:

Change $2 \frac{3}{8}$ to an improper fraction.

Step 1. Multiply the denominator of the fraction by the whole number
 $2 \times 8 = 16$.

Step 2. Add the numerator to the product of Step 1.
 $3 + 16 = 19$

Step 3. Place the sum in Step 2 over the denominator $19/8$

EXAMPLE PROBLEM:

Change $5 \frac{3}{4}$ to an improper fraction.

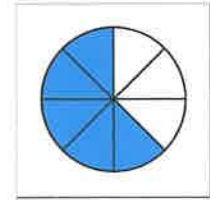
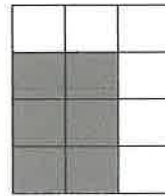
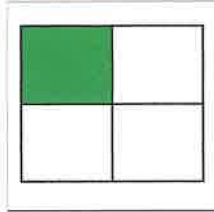
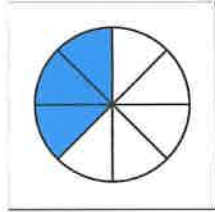
SOLUTION:

$$5 \times 4 = 20$$

$$\frac{20}{4} + 3 = \frac{23}{4}$$

INTRODUCTION TO FRACTIONS EXAMPLE PROBLEMS

PROBLEMS: What fractional part of these figures is shaded.



Ans.

3/8

(circle is divided
into 8 parts—3
are shaded)

Reduce the following fractions to their lowest terms/

$2/4 = \underline{\hspace{2cm}}$

$6/8 = \underline{\hspace{2cm}}$

$14/16 = \underline{\hspace{2cm}}$

Change the following to improper fractions.

$2 \frac{1}{2} = \underline{5/2}$

$6 \frac{3}{8} = \underline{\hspace{2cm}}$

$5 \frac{3}{4} = \underline{\hspace{2cm}}$

$(2 \times 2 = 4 + 1 = 5)$

Change the following improper fractions to mixed numbers.

$7/3 = \underline{2 \frac{1}{3}}$

$23/4 = \underline{\hspace{2cm}}$

$16/5 = \underline{\hspace{2cm}}$

$$\begin{array}{r} 2 \\ 3 \overline{)7} \\ \underline{-6} \\ 1 \end{array}$$

DECIMALS

STUDY SHEET

I. DECIMAL FRACTIONS

A. When a fraction has a denominator of 10 or the denominator is a multiple of 10, such as 100, 1,000, 10,000 etc., it is read the same as a fraction but is written as a decimal. This decimal number is called a DECIMAL FRACTION.

EXAMPLE: $\frac{3}{10}$ is read three tenths but is written .3
 $\frac{7}{100}$ is read seven hundredths but is written .07
 $\frac{9}{1,000}$ is read nine thousandths but is written .009
 $\frac{268}{10,000}$ is read two hundred sixty-eight ten thousandths but is written .0268

B. When a Decimal Fraction contains a whole number it is called a "Mixed Decimal". A mixed decimal is also read like a fraction but written as a decimal.

EXAMPLE: 24.347 is read twenty-four and three hundred forty-seven thousandths.
NOTE: Digits to the left of a decimal point are whole numbers. Digits to the right of the decimal point are fractional parts of a whole unit. Digits to the right of a decimal point are referred to as decimal places.

C. When a whole number is written by itself it is understood, but not always shown, to have a decimal point to the immediate right of the last digit of the whole number.

EXAMPLE: 7 is 7., 89 is 89., 3756 is 3756.

NOTE: Showing the decimal point at the end of a whole number does not change its value.

II. ADDITION & SUBTRACTION OF DECIMALS

A. Always remember to keep the decimal point in line vertically, even into the answer, when adding or subtracting decimals.

EXAMPLE:
$$\begin{array}{r} 3.15 \\ .076 \\ 356.75 \\ + 30.0075 \\ \hline 389.9835 \end{array}$$

$$\begin{array}{r} 2.76 \\ - 1.5 \\ \hline 1.26 \end{array}$$

B. When subtracting a decimal from a whole number you should show the decimal point in the whole number and add zeros to the right of the decimal to balance out the problem. Perform the subtraction in the same manner as with the whole numbers, keeping the decimal point in line vertically.

Example:
$$\begin{array}{r} 76. \\ - .578 \\ \hline \end{array}$$
 or
$$\begin{array}{r} 76.000 \\ - .578 \\ \hline 75.422 \end{array}$$

DECIMALS

STUDY SHEET

III. MULTIPLICATION OF DECIMALS

A. The added step is multiplying decimals, as compared to multiplying whole numbers, is the placing of the decimal point in the product (answer) correctly. The procedure to follow should be:

Step 1. Perform the multiplication in the same manner as with multiplying whole numbers.

Step 2. Count the total digits, to the right of decimal point, in both numbers being multiplied together.

Step 3. Beginning with the space between the last two digits in the product and counting each space, from right to left in the product, place the decimal point in the space that corresponds to the total digits in step two.

Step 4. Add zeros to the left of the product when there are not enough spaces.

EXAMPLE:

$$\begin{array}{r} 24.3 \\ \times 3.04 \\ \hline 972 \\ 7290 \\ \hline 73.872 \end{array}$$

total of 3
digits (decimal places)

$$\begin{array}{r} .31254 \\ \times .22 \\ \hline 62508 \\ 62508 \\ \hline 0.0687588 \end{array}$$

total of 7
decimal places

**added two zeros

NOTE: Always drop the last zero added after placing your decimal point in the product.

IV. DIVISION OF DECIMALS

A. The added step in dividing decimals, as compared to dividing whole numbers, is placing the decimal point in the quotient (answer) correctly. The procedure should be as follows:

Step 1. Move the decimal point, in the divisor, to the right, as many spaces as needed to make the divisor a whole number.

Step 2. Move the decimal point in the dividend the same number of spaces to the right as you moved in step 1. Add zeros if more spaces are required.

Step 3. Place the decimal point in the quotient directly above the new location of the decimal point in the dividend. Perform the division the same as with whole numbers.

EXAMPLE:

$$3.756 \overline{) 27.775.4}$$

$$.07 \overline{) 3.65}$$

$$5.673 \overline{) 347.600.0}$$

DECIMALS

STUDY SHEET

NOTE: In this class we will work to an accuracy of three decimal places. This means you will carry your division out four digits to the right of the decimal point and round off to three for your final answer.

V. CHANGING DECIMALS TO FRACTIONS

A. Simply convert the decimal into a fraction the same as you read it.

EXAMPLE: 0887 is read eight hundred eighty-seven thousandths.
In its fraction form it will be written $887/1,000$
.346 is $346/1,000$, .75 is $75/100$, .9 is $9/10$

B. To change a decimal to a fraction with a denominator of 16, multiply the decimal by 16. Round the quotient off to a whole number and place the whole number over 16 to form the fraction. Always reduce the fraction to its lowest terms.

EXAMPLE:

$\begin{array}{r} .7568 = 12/16 = \frac{3}{4} \\ \times 16 \\ \hline 45408 \\ \underline{7568} \\ 12.1088 \text{ round off} \end{array}$	$\begin{array}{r} .411 = 7/16 \\ \times 16 \\ \hline 2466 \\ \underline{411} \\ 6.576 \text{ round off} \end{array}$
--	--

NOTE: This procedure will work with any number desired to be the denominator. In cases of mixed decimals, multiply only the decimal by the desired denominator.

VI. CHANGING FRACTIONS TO DECIMALS

A. To change a fraction to a decimal divide the numerator by the denominator.

$$\begin{array}{r} .21875 \\ 7/32 = 32 \overline{) 7.00000} \\ \underline{64} \\ 60 \\ \underline{-32} \\ 280 \\ \underline{-256} \\ 240 \\ \underline{-224} \\ 160 \\ \underline{-160} \\ 0 \end{array}$$

NOTE: Work to an accuracy of 4 decimal places and round off to three decimal places for your final answer.

INTRODUCTION TO PERCENT

STUDY SHEET

PERCENT: Abbreviation for the Latin words "Per Centum" and means "by the hundred."

SIGN: % or ‰, thus 2% is read "TWO PERCENT" and means $2/100$ which is $2 \div 100 = .02$.

NOTICE: To write a percent as a decimal move the "decimal point" two (2) places to the LEFT (thus $2\% = .02$)

NOTE: In most calculations using percent you must first express the percent as a fraction or decimal before working with it.

EXPRESSING PERCENT AS A FRACTION OR AS A DECIMAL.

FRACTION: $12\% = 12/100$

DECIMAL: $12\% = .12$

Remembering this you should have no trouble changing percent to either a fraction or a decimal.

EXAMPLE: Express as a fraction and then as a decimal.

A. $26\% =$ _____ or _____

B. $7 \frac{1}{2} \% =$ _____ or _____

C. $250\% =$ _____ or _____

NOTICE: Don't make the common mistake of not recognizing a percent greater than 100% is greater than a single unit.

EXAMPLE: 150% is $1 \frac{1}{2}$ units.

ROUNDING OFF DECIMALS

When multiplying decimals, as in the example above, sometimes the resulting answer has too many digits to the right of the decimal point. Such a number will be of little practical use to the steamfitter-pipefitter, in certain operations such as measuring pipe. Therefore, the decimal must be reduced or "rounded off" to a number that can be worked with. This is done in the following manner:

Determine the degree of precision required for the measurement.

Look at the digit in the decimal place which indicates the required degree of precision (tenth, hundredth, thousandth)

Increase that digit by one if the digit which follows immediately is five or more and drop all digits that follow.

Leave that digit as it is if the digit which follows is less than five and drop all digits that follow.

Example: Write the decimal (4 places) .4392.
The accuracy desired is hundredths.

Step 1 Write the decimal (4 places) .4392

Step 2 Locate the digit which shows hundredths. The second digit 2, does this. .43

Step 3 Look at the *third place* digit (thousandths) to determine whether or not the second place digit should remain the same or be increased. .439

Step 4 Increase the three by one because the third place digit is greater than five.

The correct answer is .44

All of the intermediate steps in the example are given to serve as a guide in rounding off decimals. With actual practice, it is possible to

round off a decimal to any desired degree of accuracy by just looking at it.

DIVISION OF DECIMALS

We have just seen that multiplication of decimals requires a rule for the placing of the decimal points.

In the division of decimals, a similar rule is required. First of all, it is important to review the terms "dividend," and "quotient." In the example

$$\begin{array}{r} 3.3 \\ 3.6 \overline{) 11.88} \end{array}$$

the 3.6 is the divisor, the 11.88 is the dividend; and the 3.3 is the quotient or answer.

The method for correctly placing the decimal point in the answer is based on these steps:

1. First set up the problem as given:

$$3.6 \overline{) 11.88}$$

2. Then make the divisor a whole number by moving its decimal point to the right of the last figure, indicating its new position by a caret (^).

$$3.6 \wedge \overline{) 11.88}$$

3. Also move the decimal point in the dividend to the right as many places as the decimal point in the divisor has been moved. Indicate its new position by a caret (^).

$$3.6 \wedge \overline{) 11.8 \wedge 8}$$

4. When this is done, the division can be carried out as usual. Place the decimal point in the answer directly above the caret (^) in the dividend:

$$\begin{array}{r} \underline{\quad} 3.3 \\ 3.6 \wedge \overline{) 11.8 \wedge 8} \end{array}$$

Purpose:

To supply definitions of terms necessary to work with fractions, as well as to provide examples of working with fractions.

Procedure:

Study *carefully* and work the problem on the following assignment sheet.

Everyone knows about half dollars, quarters of a mile, and eighths of an inch. These are fractions. These are one or more of the equal parts of a whole, divided by the total number of equal parts of that whole. Fractions may be used to express the parts of a dollar, a mile, an inch, an apple, a circle, or anything. The first square shown in Figure 10-1 has been divided into four equal parts. In the square, the number of shaded parts (3) divided by the total number of parts (4) is the fraction of the square that is shaded. The shaded portion, then, is $\frac{3}{4}$ of the square.

NUMERATORS AND DENOMINATORS

Two numbers must be used to state a fraction. The upper number is called the "numerator" and the lower number the "denominator." The denominator indicates into how many parts the object is equally divided, while the numerator states the number of these parts under discussion.

It should be noted that, if the numerators are the same, the larger the denominator of a fraction, the smaller the value of the fraction. Thus $\frac{1}{4}$ is less than $\frac{1}{2}$, $\frac{1}{8}$ is less than $\frac{1}{2}$ or $\frac{1}{4}$ and $\frac{1}{16}$ is less than $\frac{1}{8}$, $\frac{1}{4}$ or $\frac{1}{2}$.

PROPER FRACTIONS

A fraction that is less than one ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{10}$, $\frac{1}{16}$) is called a "proper" fraction. The numerator is smaller than the denominator.

IMPROPER FRACTIONS

A fraction that is equal to one or more than one ($\frac{6}{5}$, $\frac{4}{3}$, $\frac{3}{2}$, $\frac{16}{7}$, $\frac{25}{8}$) is called an "improper" fraction. The numerator is equal to or larger than the denominator.

MIXED NUMBERS

When a number is made up of a whole number and a fraction ($1\frac{3}{4}$, $2\frac{7}{8}$), it is called a "mixed number." It represents one or a number of whole things plus part of another.

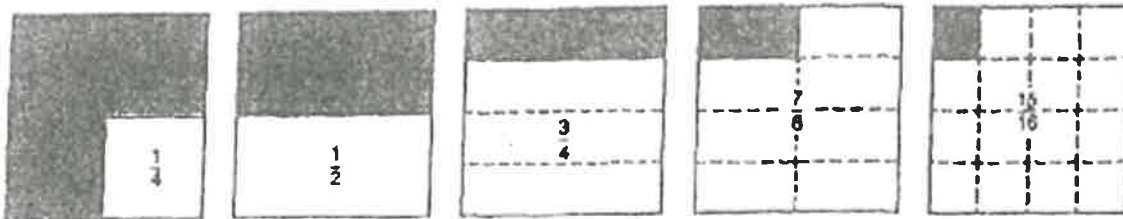


Fig. 10-1.
Fractional parts

Purpose:

To review the addition, subtraction, multiplication, and division of fractions in order to be able to work with fractions as readily as with whole numbers.

Procedure:

Study carefully and work problems on the following assignment sheet.

COMMON DENOMINATORS

A group of fractions has a common denominator when all the lower numbers of the fractions are the same. In the series of fractions $\frac{3}{16}$, $\frac{5}{16}$, $\frac{9}{16}$, and $\frac{13}{16}$, the number 16 is the common denominator.

Before a group of fractions is added or one fraction is subtracted from another, all in the group must have the same denominator.

ADDITION AND SUBTRACTION OF FRACTIONS

The addition and subtraction of fractions is simple when their denominators are alike. Just add or subtract the numerators and place over the denominator. Then reduce the fraction to its lowest terms.

Common denominators

$$\text{Addition: } \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{11}{16} + \frac{7}{16} = \frac{18}{16} = 1\frac{1}{8}$$

$$\frac{7}{18} + \frac{5}{18} = \frac{12}{18} = \frac{2}{3}$$

$$\text{Subtraction: } \frac{15}{16} - \frac{9}{16} = \frac{6}{16} = \frac{3}{8}$$

$$\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$$

$$\frac{4}{5} - \frac{2}{5} = \frac{2}{5}$$

Different denominators

Fractions to be added or subtracted usually do not have the same denominator and we first must change to a common denominator.

If there are fractions with *unlike* denominators, the fractions must be changed to equivalent fractions with like denominators before adding or subtracting. For example, the fractions $\frac{3}{8}$ and $\frac{1}{2}$ do not have common denominators. The numerator and denominator of a fraction may be multiplied by the same number without changing the value of the fraction. Thus, to add $\frac{3}{8}$ and $\frac{1}{2}$, the $\frac{1}{2}$ would have to be changed. First find by what number the 2 should be multiplied to get 8, since 8 is the least common denominator. Divide 8 by 2. The result is 4. To obtain the equivalent fraction, multiply both the numerator and the denominator by 4. The result is $\frac{4}{8}$.

Example: Add $\frac{3}{8}$ and $\frac{1}{2}$

$$\frac{3}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Addition:

$$\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$\frac{3}{8} + \frac{5}{16} = \frac{6}{16} + \frac{5}{16} = \frac{11}{16}$$

$$\frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{17}{12} = 1\frac{5}{12}$$

Subtraction:

$$\frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

$$\frac{7}{8} - \frac{1}{4} = \frac{7}{8} - \frac{2}{8} = \frac{5}{8}$$

$$\frac{15}{16} - \frac{1}{2} = \frac{15}{16} - \frac{8}{16} = \frac{7}{16}$$

LOWEST TERMS

A fraction is usually expressed in its lowest terms. It is in its lowest terms when the numerator and the denominator are such that no other number can be divided into both of them evenly, or, without a remainder. For example, the fraction $5/10$ is not in its lowest terms, since 5 can be divided into both top and bottom, thus:

$$\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

Therefore, $5/10$ in its lowest terms is $1/2$.

Fractions may be reduced to their lowest terms by dividing the numerators and denominators by the same number. For instance, to reduce $12/16$ to its lowest terms, first divide the numerator and denominator by 2.

$$\frac{12 \div 2}{16 \div 2} = \frac{6}{8}$$

This can again be divided by 2.

$$\frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

The answer in this case would have been found more quickly by dividing in the beginning by 4 instead of 2, since 4 also divides an even number of times into both 12 and 16.

$$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

A correct solution to any problem requires that all fractions be in their lowest terms.

CHANGING MIXED NUMBERS TO IMPROPER FRACTIONS

Multiplication and division require the changing of mixed numbers to improper fractions. Figure 10-2 shows a mixed number, $2 \frac{3}{8}$, and the equivalent improper fraction, $19/8$. There are three steps in changing a mixed number to an improper fraction.

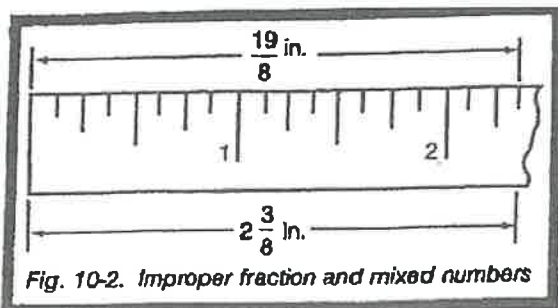


Fig. 10-2. Improper fraction and mixed numbers

Example: Change $2 \frac{3}{8}$ to an improper fraction:

1. Multiply the denominator of the fraction by the whole number:

$$8 \times 2 = 16$$

2. Add the numerator to the result:

$$3 + 16 = 19$$

3. Place this result over the denominator:

$$\frac{19}{8}$$

Example Problem:

Change $5 \frac{3}{4}$ to an improper fraction.

Solution: $5 \times 4 = 20$

$$\frac{20}{4} + \frac{3}{4} = \frac{23}{4}$$

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

Improper fractions usually result from the addition, subtraction, multiplication, or division of fractions. For example, after adding half dollars or eighths of inches, the totals may be $7/2$ dollars or $13/8$ inches. However, we do not say seven half dollars or thirteen eighths' inches. Instead the improper fractions are changed to mixed numbers: $3 \frac{1}{2}$ dollars or $1 \frac{5}{8}$ inches. This change is easily made by dividing the numerator by the denominator, using ordinary long or short division.

For instance, by long division, change the improper fraction $1 \frac{3}{8}$ to a mixed number.

$$\begin{array}{r} 1 \\ 8 \overline{)13} \\ \underline{8} \\ 5 \end{array} = 1 \frac{5}{8}$$

By short division, it would be

$$\frac{13}{8} = 13 \div 8 = 1 \frac{5}{8}$$

ADDITION AND SUBTRACTION OF MIXED NUMBERS

We have already seen how to change fractions to a common denominator, in order to add or subtract them. The same method applied to the fractional parts of mixed numbers. To add and subtract mixed numbers it is necessary to work first with the fractions and then with the whole numbers.

Addition:

$$4\frac{3}{4} + 2\frac{1}{8} = 4\frac{6}{8} + 2\frac{1}{8} = 6\frac{7}{8}$$

$$5\frac{5}{8} + 9\frac{11}{16} = 5\frac{10}{16} + 9\frac{11}{16} = 14\frac{21}{16} = 15\frac{5}{16}$$

Subtraction:

$$2\frac{1}{2} - 1\frac{1}{4} = 2\frac{2}{4} - 1\frac{1}{4} = 1\frac{1}{4}$$

In the subtraction of mixed numbers it is sometimes necessary to borrow from the whole number. This will provide a fraction large enough to complete the subtraction.

$$3\frac{1}{6} - 1\frac{1}{3} = 2\frac{7}{6} - 1\frac{2}{6} = 1\frac{5}{6}$$

$$4 - 2\frac{7}{8} = 3\frac{8}{8} - 2\frac{7}{8} = 1\frac{1}{8}$$

$$5\frac{1}{8} - 2\frac{9}{16} = 5\frac{2}{16} - 2\frac{9}{16} = 4\frac{18}{16} - 2\frac{9}{16} = 2\frac{9}{16}$$

MULTIPLICATION OF FRACTIONS AND MIXED NUMBERS

Multiplying fractions is simpler than adding or subtracting them. It is not necessary to have a common denominator.

To multiply two fractions, simply multiply the top numbers together. This is the numerator. Then multiply the bottom numbers together. This is the denominator. Then, reduce all answers to lowest terms as illustrated below. For instance, multiply $\frac{3}{4} \times \frac{2}{3}$.

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$$

Whenever a mixed number is to be multiplied, change it to a fractional form and proceed as with a fraction:

$$2\frac{3}{4} \times 4\frac{1}{2} = \frac{11}{4} \times \frac{9}{2} = \frac{99}{8} = 12\frac{3}{8}$$

To multiply a fraction and a whole number, treat the whole number as if it had a 1 under it. For instance:

$$6 = \frac{6}{1}$$

$$7 = \frac{7}{1}$$

$$99 = \frac{99}{1}$$

Then proceed as follows:

$$3 \times \frac{5}{8} = \frac{3 \times 5}{1 \times 8} = \frac{15}{8} = 1\frac{7}{8}$$

This procedure for multiplying two fractions or mixed numbers will do as well for more than two numbers. No new methods are necessary for such work as

$$\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{3 \times 2 \times 1}{4 \times 3 \times 2} = \frac{6}{24} = \frac{1}{4}$$

CANCELLATION

Cancellation is a technique, in multiplying, for cutting down and simplifying our work. This is done by using smaller numbers rather than larger ones. By cancelling, we reduce to lowest terms, so far as possible, before multiplying, rather than afterwards. Take the following problem, for example, and multiply it out:

$$\frac{3}{4} \times \frac{4}{7} \times \frac{5}{3} = \frac{\cancel{3} \times \cancel{4} \times 5}{4 \times 7 \times \cancel{3}} = \frac{60}{84} = \frac{5}{7}$$

On the other hand, if it is known that the 3 and 4 in the numerator would cancel the 3 and 4 in the denominator, it would be written thus:

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{4}} \times 5}{\cancel{4} \times 7 \times \cancel{3}} = \frac{5}{7}$$

Cancelling can also be done by numbers that do not appear in either the numerator or denominator; that is, any number may be divided into one of the numbers in the numerator and one of the numbers in the denominator, provided there is no remainder. To cancel out.

$$\frac{4}{5} \times \frac{1}{6} = \frac{4 \times 1}{5 \times 6} =$$

Notice that the number 2 will divide evenly into both the 4 of the numerator and the 6 of the denominator. Cancel these numbers and write in 2 and 3:

$$\frac{\overset{2}{\cancel{4}} \times 1}{5 \times \underset{3}{\cancel{6}}} = \frac{2}{15}$$

Here are steps in three examples. The steps make it easy to shorten the work by cancellation.

1. In the example

$$\frac{3 \times 4 \times 7}{4 \times 7 \times 3}$$

We shall cancel the 3 in the numerator and denominator:

$$\frac{\overset{1}{\cancel{3}} \times 4 \times 7}{4 \times 7 \times \underset{1}{\cancel{3}}}$$

Likewise, cancel the 4 and 7 in the numerator and denominator:

$$\frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{4}} \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{3}}} = \frac{1}{1} = 1$$

2. Multiply

$$\frac{5}{6} \times \frac{3}{10} \times \frac{1}{2}$$

In this case first divide by 5

$$\frac{\overset{1}{\cancel{5}} \times 3 \times 1}{6 \times \overset{1}{\cancel{10}} \times 2}$$

and then by 3

$$\frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{3}} \times 1}{\underset{2}{\cancel{6}} \times \overset{1}{\cancel{10}} \times 2} = \frac{1}{8}$$

3. Multiply

$$\frac{6}{7} \times \frac{14}{15} \times \frac{3}{4} = \frac{\overset{3}{\cancel{6}} \times \overset{2}{\cancel{14}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{7}} \times \underset{5}{\cancel{15}} \times \underset{2}{\cancel{4}}} = \frac{3}{5}$$

Note: Cancellation does not apply in addition or subtraction.

DIVISION OF FRACTIONS AND MIXED NUMBERS

As compared with the multiplication of fractions, one additional step is needed for the division of fractions. The rule for division of fractions is to invert the divisor (what we are dividing by) and multiply. To invert a fraction means to turn it upside down. $\frac{3}{5}$ inverted is $\frac{5}{3}$. For instance, divide $\frac{3}{4}$ by $\frac{1}{2}$.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

As in multiplication, mixed numbers must be changed to fractions. This must be done before inverting the divisor. Likewise, a whole number divisor must have a 1 placed under it before inverting: $5 = \frac{5}{1}$; inverted it is $\frac{1}{5}$. To divide $4\frac{2}{3}$ by $1\frac{1}{5}$.

$$4\frac{2}{3} \div 1\frac{1}{5} = \frac{14}{3} \div \frac{6}{5} = \frac{14}{3} \times \frac{5}{6} = \frac{56}{9} = 6\frac{2}{9}$$

After the divisor has been inverted we have a multiplication problem and may cancel where possible. Before starting the division calculations, the divisor must be properly selected. It should follow the division sign (\div) in the equation. Do not invert the figures before the divisor.